



Univerza v Mariboru

Fakulteta za naravoslovje  
in matematiko

UČNI NAČRT PREDMETA / COURSE SYLLABUS						
<b>Predmet:</b>	<b>Analiza IV</b>					
<b>Course title:</b>	<b>Analysis IV</b>					
<b>Študijski program in stopnja</b> Study programme and level	<b>Študijska smer</b> Study field			<b>Letnik</b> Academic year	<b>Semester</b> Semester	
Matematika				3.	5.	
Mathematics				3.	5.	
<b>Vrsta predmeta / Course type</b>				Obvezni / Obligatory		
<b>Univerzitetna koda predmeta / University course code:</b>						
<b>Predavanja</b> Lectures	<b>Seminar</b> Seminar	<b>Sem. vaje</b> Tutorial	<b>Lab. vaje</b> Laboratory work	<b>Teren. vaje</b> Field work	<b>Samost. delo</b> Individ. work	<b>ECTS</b>
60		45			135	8
<b>Nosilec predmeta / Lecturer:</b>				Bojan HVALA		
<b>Jeziki / Languages:</b>	<b>Predavanja / Lectures:</b>	SLOVENSKO/SLOVENE				
	<b>Vaje / Tutorial:</b>	SLOVENSKO/SLOVENE				
<b>Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:</b>				<b>Prerequisites:</b>		
Jih ni.				There are none.		
<b>Vsebina:</b>				<b>Content (Syllabus outline):</b>		
Dvojni in trojni integral. Polarne, cilindrične in sferne koordinate. Uporaba. Posplošeni dvojni integral.				Double and triple integrals. Polar, cylindrical and spherical coordinates. Applications. Generalized double integral.		
Krivulje v ravnini in prostoru. Parametrizacija, tangenta, ločna dolžina, ukrivljenost. Spremljajoči trieder. Ploskve v prostoru. Površina.				Plane and space curves. Parametrization, tangent, arc length, curvature. Surfaces. Area.		
Skalarna in vektorska polja. Krivuljni in ploskovni integrali. Gradient, divergenca, rotor.				Scalar and vector fields. Line and surface integrals. Gradient, divergence, curl.		
Gaussov in Stokesov izrek. Primeri uporabe.				Divergence theorem. Stokes' theorem. Applications.		



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### Temeljni literatura in viri / Readings:

- M. H. Protter, C. B. Morrey: *Intermediate calculus*. New York : Springer, 1985.
- D. Varberg, E. Purcell, S. Rigdon: *Calculus*. Prentice Hall, 2006.
- S. Lang: *Calculus of several variables*, Reading: Addison – Wesley, 1973.
- D. J. Struik: *Lectures on Classical Differential Geometr.* Cambridge: Mass., Addison-Wesley Press, 1950.
- M. Dobovišek: *Rešene naloge iz analize II*. Ljubljana: DMFA, 1996.
- B. Hvala: *Zbirka izpitnih nalog iz analize*. Ljubljana: DMFA, 1996.

### Cilji in kompetence:

Spoznati glavna dejstva o krivuljah in ploskvah.  
Spoznati koncepte integracije po merljivih množicah v ravnini in prostoru ter po krivuljah in ploskvah. Spoznati povezave med obravnavanimi integrali. Seznaniti se z možnostmi uporabe te teorije v fiziki in drugje.

### Objectives and competences:

Knowing basic facts about curves and surfaces. Knowing concepts of integration on measurable subsets of plane and space and on curves and surfaces. Knowing basic relations between those integrals. Being familiar with the applications of this theory.

### Predvideni študijski rezultati:

Znanje in razumevanje:

Po zaključku tega predmeta bo študent

- Razumel koncept dvojnega in trojnega integrala, znal te integrale računati in jih uporabljati v standardnih situacijah.
- Poznal osnove diferencialne geometrije krivulj in ploskev v prostoru. Poznal bo definicije dolžine loka, ukrivljenosti krivulje, površine ploskve in jih znal računati.
- Znal definirati integrale skalarnih in vektorskih funkcij po krivuljah in ploskvah, poznal praktično ozadje teh definicij in jih znal računati. Poznal bo rezultate, ki te integrale povezujejo in jih znal uporabiti.

Prenesljive/ključne spretnosti in drugi atributi:

- *Reševanje problemov*: Širjenje zavesti o moči matematike pri opisovanju sveta.

### Intended learning outcomes:

Knowledge and Understanding:

On completion of this course a student will be able to

- Understand the concept of double and triple integrals, be able to calculate these integrals and use them in standard situations.
- Recall the basics of the differential geometry of the curves and surfaces in the space. He will be able to recall the definitions of the length of the arc, the curvature of the curve and the area of a surface. He will be able to compute them.
- Define curve and surface integrals of scalar and vector fields, understand the practical background of these definitions and be able to compute them. He will be able to recall the results that connect these integrals and apply them.

Transferable/Key Skills and other attributes:

- *Problem solving*: Spreading awareness of the power of mathematics in describing the world. Use of discussed topics in geography,



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<p>Uporaba obravnavanih tematik v geografiji, astronomiji, fiziki.</p> <ul style="list-style-type: none"><li>• <i>Tehnične spretnosti</i>: Uporaba računalnika pri predstavitvi ploskev in krivulj ter pri razumevanju ozadja Jacobijeve determinante.</li></ul>	<p>astronomy, physics.</p> <ul style="list-style-type: none"><li>• <i>Technical skills</i>: Using the computer in presenting surfaces and curves, and in explaining the background of Jacobi's determinant.</li></ul>
<b>Metode poučevanja in učenja:</b>	<b>Learning and teaching methods:</b>
<ul style="list-style-type: none"><li>• Predavanja</li><li>• Teoretične vaje</li></ul>	<ul style="list-style-type: none"><li>• Lectures</li><li>• Theoretical exercises</li></ul>
<b>Načini ocenjevanja:</b>	<b>Assessment:</b>
<p><u>Izpit:</u> Pisni izpit – problemi Ustni izpit</p> <p>Vsaka izmed naštetih obveznosti mora biti opravljena s pozitivno oceno.</p> <p>Opravljen pisni izpit – problemi je pogoj za pristop k ustnemu izpitu.</p> <p>Pisni izpit – problemi se lahko nadomesti z dvema delnima testoma (sprotne obveznosti).</p>	<p><u>Delež (v %) / Weight (in %)</u></p> <p>50% 50%</p> <p><u>Exams:</u> Written exam – problems Oral exam</p> <p>Each of the mentioned assessments must be assessed with a passing grade.</p> <p>Passing grade of written exam – problems is required to take the oral exam.</p> <p>Written exam – problems can be replaced with two mid-term tests.</p>

**Reference nosilca / Lecturer's references:**

1. HVALA, Bojan. Cevian cousins of a triangle centroid. *Journal for geometry and graphics*, ISSN 1433-8157, 2015, vol. 19, no. 2, str. 211-218. [COBISS.SI-ID [22024200](#)]
2. HVALA, Bojan. A generalized Seebach's theorem. *Beiträge zur Algebra und Geometrie*, ISSN 0138-4821, 2014, vol. 55, iss. 2, str. 471-478, doi: [10.1007/s13366-013-0165-3](https://doi.org/10.1007/s13366-013-0165-3). [COBISS.SI-ID [20843272](#)]
3. HVALA, Bojan. Značilne točke trikotnika kot funkcije. *Obzornik za matematiko in fiziko*, ISSN 0473-7466, 2014, letn. 61, št. 1, str. 1-14. [COBISS.SI-ID 16937817].
4. HVALA, Bojan. Diophantine Steiner triples and Pythagorean-type triangles. *Forum geometricorum*, ISSN 1534-1178, 2010, vol. 10, str. 93-97. <http://forumgeom.fau.edu/FG2010volume10/FG201010.pdf>. [COBISS.SI-ID [15669337](#)]
5. HVALA, Bojan. Modernizing mathematics education in Slovenia : a teacher friendly approach.



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V: LAMANAUSKAS, Vincentas (ur.). *Challenges of science, mathematics and technology teacher education in Slovenia*, (Problems of education in the 21st century, ISSN 1822-7864, vol. 14). Siauliai: Scientific Methodological Center Scientia Educologica. 2009, str. 34-43. [COBISS.SI-ID [17351944](#)]