Univerza v Mariboru

Fakulteta za naravoslovje in matematiko

UČNI NAČRT PREDMETA / COURSE SYLLABUS	
Predmet:	**Matematično modeliranje**
Course title:	Mathematical Modelling

Študijski program in stopnja Study programme and level	**Študijska smer Study field**	**Letnik Academic year**	**Semester Semester**
Enovit magistrski študijski program druge stopnje Predmetni učitelj	**/**	**4.**	**8.**
Five-year master's degree program Subject Teacher	**/**	**4.**	**8.**

Vrsta predmeta / Course type	Obvezni / Obligatory
Univerzitetna koda predmeta / University course code:	

Predavanja Lectures	**Seminar Seminar**	**Sem. vaje Tutorial**	**Lab. vaje Laboratory work**	**Teren. vaje Field work**	**Samost. delo Individ. work**	**ECTS**
30	**15**	**-**	**15**	**-**	**90**	**5**

Nosilec predmeta / Lecturer:	Drago Bokal

Jeziki / Languages:		
	Predavanja / Lectures:	Slovensko/Slovene ali Angleško/English
	Vaje / Tutorial:	Slovensko/Slovene ali Angleško/English

Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:	**Prerequisits:**
Poznavanje enostavnih algoritmov. Poznavanje osnov linearne algebre in vektorske analize.	Knowledge of simple algorithms. Knowledge of basic linear algebra and calculus.

Vsebina:	**Content (Syllabus outline):**
Obvezna vsebina, ki pri študentih vzpostavi temeljni nabor znanj s področja operacijskih raziskav:	Mandatory content that familiarizes the students with fundamentals of mathematical modeling:

● Pregled vrst matematičnih modelov. Proces izdelave matematičnega modela. Vrste spremenljivk. ● Matematično modeliranje in inovacijski proces. ● Matematični modeli znanja in učenja: spodbujevano učenje, vmesniška teorija zaznav, univerzalni model procesa. ● Modeliranje procesov. Sekvenčni diagrami, diagrami poteka, Petrijeve mreže. ● Podatkovno modeliranje. Entitetni diagram, diagram razredov. ● Modeliranje odločitev, odločitveno drevo, modelsko drevo, drevo igre, acikličen odločitven proces. ● Linearni program. Dual. Farkaseva lema. Senčne cene. Analiza občutljivosti. ● Analiza podatkov, verjetnost, Monte Carlo simulacija. V okviru obvezne vsebine študentje izdelajo seminarsko nalogo, z dvema vmesnima in zaključno predstavitvijo, preko katerih utrdijo poznavanje procesa matematičnega modeliranja. Naloge so povezane z njihovo bodočo kariero (praktični problemi iz gospodarstva, teoretični problemi iz teorije modeliranja, optimizacije, algoritmov). Preostala predavanja se prilagodijo projektom, ki so jih izbrali študentje, in obsegajo naslednje vsebine: ● Uvod v teorijo iger. Nashevo ravnovesje. Matrične igre z ničelno vsoto. Optimizacijski modeli s centraliziranim odločanjem, modeli teorije iger s porazdeljenim odločanjem. ● Simulacijski modeli. Modeliranje sprememb z diferenčnimi in diferencialnimi enačbami. ● Matematično obnašanje dinamičnih sistemov. ● Uporaba matematičnih modelov v znanosti in tehniki.. ● Relacijski modeli na primeru urnika in razvrščanja nalog. ● Deterministični, stohastični, robustni problemi. Stohastični linearni program	● Overview of mathematical model types. Process of mathematical model creation. Variable types. ● Mathematical model and inovation process. ● Mathematical models of knowledge and learning: reinforcement learning, interface theory of perception, universal process model. ● Process modelling. Sequence diagrams, flow charts, Petri nets. ● Data modelling. Entity relationship diagrams, Class diagrams. ● Decision modeling. Decision tree. Regression tree, game tree, acyclic decision process. ● Linear program and its dual. Farkash lema. Shadow prices. Sensitivity analysis. ● Data analysis, probability, Monte Carlo simulations. Within the coursework, the students select problems whose result is a coursework report that is presented in two intermediate and one final presentation in front of the class. The problems are related to their future career (practical problems from industry and business, theoretical problems from the areas of optimization, algorithms, modelling). The content of the remaining lectures is selected according to these projects from the following list: ● Introduction to game theory. Nash equilibria. Matrix zero sum games. Optimization models with centralized decision making. Game theory models with distributed decision making. ● Simulation models. Modeling changes with diference and diferential equations. ● Mathematical behaviour of dynamic systems. ● Applications of mathematical models in science and engineering. ● Relational models applied to timetabling and scheduling problems. ● Deterministic, stohastic, robust problems. Stohastic linear program

(diskretna spremenljivka). Dekompozicija. Deterministični in stohastični modeli optimizacije portfelja. ● Problem prehrane. Simpleksna metoda. ● Aplikacije teorije iger: optimalna strategija na tržišču z dvema konkurentoma. ● Čakalne vrste. ● Druge vsebine s področja matematičnega modeliranja, povezane s študentskimi projekti. V okviru vsebin so predstavljene tudi odprtokodne in komercialne tehnološke rešitve za obravnavo navedenih modelov. Za organizacijo, predstavitev in analizo podatkov se uporablja Excel. Linearni programi se rešujejo z reševalci v tehnologiji Python, R, AMPL, Matlab, odvisno od okolja, iz katerega izhaja študentov problem.	(discrete variable). Decomposition. Deterministic and stohastic models of portfolio optimization. ● Diet problem. Simplex method. ● Applications of game theory: optimal strategy in two competitor market. ● Queues. ● Other material from the field of mathematical modeling, related to students' projects. The students are familiarized with open-source and commercial technological solutions for treatment of the studied mathematical models. Excel is used for initial data organization, presentation, and analysis. Students are introduced to different linear programming solvers: Python, R, AMPL, Matlab, depending on the environment the student's problem is coming from.

Temeljni literatura in viri / Readings:

Osnovno / basic:

● Osais, Yahya Esmail. *Computer Simulation: A Foundational Approach Using Python*. Chapman and Hall/CRC, 2017.

● R. Rardin. Optimization in Operations Research. Prentice Hall, Inc., Upper Saddle River, New Jersey, 2000.

● J. Franklin, Methods of Mathematical Economics: Linear and Nonlinear Programming, Fixed-Point Theorems. Classics in Applied Mathematics 37, SIAM, 2002.

● Dossey, Giordano, McCrone, Weir, Mathematics Methods and Modelling for today's Mathematics Classroom, Brooks/Cole, Pacific Grove, 2002.

Dodatno / additional:

● E. Zakrajšek, *Matematično modeliranje,* DMFA – Založništvo, Ljubljana, 2004.

● J.D. Murray, *Mathematical biology I. An introduction*, Springer,New York, 2002.

● G. Polya, *Kako rešujemo matematične probleme*, DMFA, 1989.

Cilji in kompetence:	Objectives and competences:
● Spoznati osnovne tehnike in prijeme matematičnega modeliranja po principih snovalskega razmišljanja. ● Spoznati teoretična ozadja matematičnega modeliranja in vpetost v druge znanstvene discipline. ● Uporabiti osnovne algoritme in hevristike za reševanje matematičnih problemov.	● Understanding of basic techniques of mathematical modeling using principles of design thinking. ● Acquaintance with the theoretical background of mathematical modeling. ● Find and apply technological tools to develop and investigate a mathematical model. ● Understanding of basic applications of algorithms and heuristics to solve mathematical problems.

● Najti in uporabiti tehnološka orodja za izdelavo in raziskovanje matematičnega modela. ● Uporabiti znanje drugih matematičnih predmetov pri analizi praktičnih problemov. ● Pridobiti izkušnje pri izdelavi matematičnega modela, uporabnega pri kasnejši karieri. ● Pridobiti kompetenco iskanja in povzemanja literature o problemih, ki jih srečamo pri obravnavi modela. ● Seznaniti se z načini prepoznavanja za model pomembnih podatkov o problemu. ● Pridobiti izkušnje pri pojasnjevanju matematičnega modela in zagovarjanju njegovih predpostavk.	● Apply the knowledge from other mathematical areas in analysis of practical problems. ● Gain experience in developing a mathematical model, useful in their future career. ● Learn about sources of bibliography on problems related to studying mathematical models. ● Learn to distinguish the relevant data for the model under study. ● Gain experience in explaining and presenting the mathematical model and defending its assumptions.

Predvideni študijski rezultati:	**Intended learning outcomes:**
Znanje in razumevanje: ● Poglobiti znanje iz matematičnih metod optimizacije. ● Poglobiti znanje iz uporabe matematičnega modeliranja v znanosti in praksi. ● Podrobno poglobi znanje iz zahtevnejših aplikacij matematičnega modeliranja v finančni optimizaciji. Prenesljive/ključne spretnosti in drugi atributi: ● Direktne aplikacije v finančni matematiki, ekonomiji, poslovnih vedah, inžinirstvu, kemiji in številnih drugih družboslovnih in naravoslovnih vedah. Obenem principi linearne optimizacije tvorijo osnovo za matematično programiranje. ● Izdelava temeljitega elaborata ali zgoščenega članka uporabe matematičnega modela na konkretnem matematičnem problemu, ki ga študent najde ob razmisleku o svoji bodoči karieri.	Knowledge and Understanding: ● To deepen the knowledge of mathematical methods of optimization. ● To deepen the knowledge of applications of mathematical modelling in research and practice. ● To deepen the knowledge of details of advanced applications of mathematical modelling in financial optimization. Transferable/Key Skills and other attributes: ● Direct applications in finacial mathematics, economy, business, engineering, chemistry, and numerous other social and natural sciences. Also, principles of linear optimization are foundations for mathematical programming. ● Preparation of a detailed technical report or focused report paper describing a mathematical model of a specific mathematical problem the students encounter while investigation their possible future careers.

Metode poučevanja in učenja:	**Learning and teaching methods:**

● Na predavanjih študentje spoznajo predpisano snov predmeta. Z uporabo obrnjenega (flipped) poučevanja na predavanjih aktivno spoznavajo povezavo med snovjo in njihovimi projekti. ● V okviru seminarskih vaj študentje razumevanje snovi utrjujejo na projektih, povezanih z njihovo bodočo kariero. Razporejeni so v manjše skupine, ki po metodah projektnega učenja delajo na izbranih projektih. ● V okviru seminarja študentje uporabljajo, analizirajo in vrednotijo svoje projekte za potrebe ustvarjanja novih rešitev za okolje, iz katerega izvirajo problemi. ● Z uporabo izkušanja matematičnega modeliranja študenti merijo svoj napredek pri predmetu, ga reflektirajo in tako uporabljajo njegov matematični model za opazovanje izboljšanja, ki izvira iz matematično podprtega odločanja. ● Tri predstavitve njihovih rezultatov pomagajo študentom priučiti se suverenosti javnega nastopanja in zagovarjanja doseženih rezultatov. Osredotočeni dvodnevni organizirani hackathon pomaga študentom izkusiti veselje ob osredotočenem raziskovanju in sodelovanju z vrstniki.	● At the lectures, the students are familiarized with the required contents of the course. Applying flipped learning approach, they discuss their coursework projects in relation to the material of the course. ● Within the coursework, the students deepen their understanding of the material on projects, related to their future careers. They are organized in smaller groups who apply the principles of project based learning on three smaller projects. ● At the seminar, the students apply, analyse, and evaluate their projects in order to create new solutions desired by the environment the problems are coming from. ● Applying embedded mathematical modelling, the students measure and reflect upon their progress thus using its mathematical model to experience improvement coming from mathematically supported decision making. ● Three presentations of their results help students acquire confidence with public presentation and defending their results. ● Focused two-day workshop-style organized hackathon helps students experience the joy of focused research and collaboration with peers.
Načini ocenjevanja:	**Assessment:**

Način (ustno izpraševanje, seminarska naloga - projekt):	Delež (v %) / Weight (in %)	Type (oral examination, coursework - project):
Seminarska naloga s tremi predstavitvami, ca. 90 ur samostojnega dela.	75%	Coursework report, approx. 90 hours of individual work.
Ustni izpit	25%	Oral exam
Vsaka izmed naštetih obveznosti mora biti opravljena s pozitivno oceno.		Each of the mentioned commitments must be assessed with a passing grade.
Pozitivna ocena pri seminarski nalogi je pogoj za pristop k izpitu.		Passing grade of the coursework report is required for taking the exam.

Reference nosilca / Lecturer's references:

Glej COBISS/SICRIS. http://sicris.izum.si/search/rsr.aspx?lang=slv&id=15413
SMOLE, Andreja, JAGRIČ, Timotej, BOKAL, Drago. Principal/Two-Agent model with internal signal. *Central European journal of operations research*. sep. 2021, vol. 29, no. 3, str. 791-808, ilustr. ISSN 1435-246X. https://link.springer.com/content/pdf/10.1007/s10100-020-00719-0.pdf, DOI: 10.1007/s10100-020-00719-0. [COBISS.SI-ID 75068163]

BOKAL, Drago, STEINBACHER, Mitja. Phases of psychologically optimal learning experience : task-based time allocation model. Central European Journal of Operations Research, ISSN 1435-246X, 2019, str. 1-23, doi: 10.1007/s10100-019-00609-0. [COBISS.SI-ID 24408328]
BOKAL, Drago, BREŠAR, Boštjan, JEREBIC, Janja. A generalization of Hungarian method and Hall's theorem with applications in wireless sensor networks. Discrete Applied Mathematics, ISSN 0166-218X. [Print ed.], 2012, vol. 160, iss. 4-5, str. 460-470. http://dx.doi.org/10.1016/j.dam.2011.11.007. [COBISS.SI-ID 16191577]

BOKAL, Drago, DEVOS, Matt, KLAVŽAR, Sandi, MIMOTO, Aki, MOOERS, Arne Ø. Computing quadratic entropy in evolutionary trees. Computers & Mathematics with Applications, ISSN 0898-1221. [Print ed.], 2011, vol. 62, no. 10, str. 3821-3828. http://dx.doi.org/10.1016/j.camwa.2011.09.030. [COBISS.SI-ID 16059481]VEGI KALAMAR, Alen, ŽERAK, Tadej, BOKAL, Drago. Counting Hamiltonian cycles in 2-tiled graphs. *Mathematics*. 2021, vol. 9, iss. 6, str. 1-27. ISSN 2227-7390. DOI: 10.3390/math9060693. [COBISS.SI-ID 61574403]