



Univerza v Mariboru

Fakulteta za naravoslovje  
in matematiko

### UČNI NAČRT PREDMETA / COURSE SYLLABUS

<b>Predmet:</b>	<b>Kombinatorična optimizacija</b>
<b>Course title:</b>	<b>Combinatorial Optimization</b>

Študijski program in stopnja Study programme and level	Študijska smer Study field	Letnik Academic year	Semester Semester
Matematika 2. stopnja		1. ali 2.	1. ali 3.
Mathematics 2 <sup>nd</sup> cycle		1. or 2.	1. or 3.

**Vrsta predmeta / Course type**

**Univerzitetna koda predmeta / University course code:**

Predavanja Lectures	Seminar Seminar	Sem. vaje Tutorial	Lab. vaje Laboratory work	Teren. vaje Field work	Samost. delo Individ. work	ECTS
30	15	30			135	7

**Nosilec predmeta / Lecturer:**

**Jeziki / Languages:**  
**Predavanja / Lectures:**   
**Vaje / Tutorial:**

**Pogoji za vključitev v delo oz. za opravljanje študijskih obveznosti:**

**Prerequisites:**

**Vsebina:**  
Obvezna vsebina, ki pri študentih vzpostavi temeljni nabor znanj s področja kombinatorične optimizacije:

- Večkriterijska linearna optimizacija. Ciljno programiranje. Celoštevilsko programiranje.
- Problem nahrbtnika in njegove različice.
- Pretoki v omrežjih. Ford-Fulkersonov algoritem.
- Problem maksimalnega prirejanja.

**Content (Syllabus outline):**  
Mandatory content, that familiarizes the students with fundamentals of operations research and mathematical programs:

- Multicriteria linear optimization. Goal programming. Integer programming.
- Knapsack problems and its variants.
- Network flows. Ford-Fulkerson's algorithm.
- Maximum matching problem.
- Maximum cut problem.

- Problem maksimalnega prereza.
- Transportni problem. Problem kitajskega poštarja. Problem trgovskega potnika.
- Aproksimacijski algoritmi.
- Hevristike in metahevristike. Lokalna optimizacija. Tabu search. Simulirano ohlajanje. Genetski algoritmi. Nevronske mreže. Samo-organizirajoče se mreže.

V okviru vsebine študentje izberejo zahtevnejši problem - projekt, s katerimi se poglobljeno ukvarjajo pri seminarski nalogi. Problem je povezan z njihovo bodočo kariero (praktični problemi iz gospodarstva, teoretični problemi iz teorije matematičnega programiranja in pripadajočih numeričnih algoritmov). Preostala predavanja se prilagodijo problemom, ki so jih izbrali študentje, in obsegajo vsebine z naslednjega seznama:

- Optimalni portfelj celoštevilskih lotov in celoštevilsko programiranje.
- Problem delovnega zaporedja.
- Problem urnika.
- Problem razporejanja nalog enega in več strojev.
- Problem optimizacije zalog.
- Problemi rezanja in pakiranja.
- Drugi s kombinatorično optimizacijo povezani problemi, usklajeni s študentskimi interesi.

- Transport problem. Chinese postman problem.
- Travelling salesman problem.
- Approximation algorithms.
- Heuristics and metaheuristics. Local optimization. Tabu search. Simulated annealing. Genetic algorithms. Neural nets. Self-organized maps.

Within the coursework, the students select a problem - project whose result is coursework report. The problem is related to their future career (practical problems from industry and business, theoretical problems from the areas of optimization, algorithms, modelling). The content of the remaining lectures is selected according to these projects from the following list:

- Optimal portfolio of integer lots and integer programming.
- Employee timetabling problem.
- School timetabling problems.
- Scheduling tasks of one or several processors.
- Stock optimization.
- Cutting and packing problems.
- Other problems, related to combinatorial optimization and student's interest.

### Temeljni literatura in viri / Readings:

J.Žerovnik: Osnove teorije grafov in diskretne optimizacije (druga izdaja), Fakulteta za strojništvo, Maribor 2005.

R. Wilson, M. Watkins, Uvod v teorijo grafov, DMFA, Ljubljana 1997.

B. Robič: Aproksimacijski algoritmi, Založba FRI, Ljubljana 2002.

E. Zakrajšek: Matematično modeliranje, DMFA, Ljubljana 2004.

B. Korte, J. Vygen: Combinatorial Optimization, Theory and Algorithms, Springer, Berlin 2000.

D. Cvetković, V. Kovačević-Vujčić: Kombinatorna optimizacija, DOPIS Beograd 1996.

S. Zlobec, J. Petrić: Nelinearno programiranje, Naučna knjiga, Beograd 1989.

E. Kreyszig: Advanced Engineering Mathematics, (seventh edition), Wiley, New York 1993.

### Cilji in kompetence:

Usvojiti proces matematičnega modeliranja na diskretnih optimizacijskih problemih.

Razviti kompetenco samostojnega apliciranja matematičnih metod na probleme iz finančne

### Objectives and competences:

Familiarize the students with the process of mathematical modelling of continuous optimization problems.

Develop competent skills of independent

optimizacije, ekonomije, ter širše iz gospodarstva.

Spoznati tehnološka orodja, s katerimi se srečujemo pri reševanju optimizacijskih problemov in problemov matematičnega modeliranja.

application of mathematical methods to the problems from financial optimization, economics, and broader from industry.

Familiarize the students with technological tools that assist solving optimization problems and problems related to mathematical modelling.

### **Predvideni študijski rezultati:**

Znanje in razumevanje:

- Razumevanje zahtevnejših principov kombinatorične optimizacije.
- Poglobi znanje iz sodobnih metod za reševanje problemov kombinatorične optimizacije.
- Poglobiti znanje iz diskretnih modelov in drugih zahtevnih aplikacij kombinatorične optimizacije v finančni matematiki, optimiranju virov, in širše

Prenesljive/ključne spretnosti in drugi atributi:

- Direktne aplikacije v finančni matematiki, ekonomiji, poslovnih vedah, inženirstvu, fiziki in številnih drugih družboslovnih in naravoslovnih vedah.
- Suvereno obvladovanje procesa matematičnega modeliranja in uporabe tehnik kombinatorične optimizacije v problemih s področja finančne optimizacije, ekonomije in širše.
- Izdelava temeljitega elaborata ali zgoščenega članka uporabe matematičnega modela na konkretnem matematičnem problemu, ki ga študent najde ob razmisleku o svoji bodoči karieri.

### **Intended learning outcomes:**

Knowledge and Understanding:

- To be able to understand advanced principles of combinatorial optimization.
- To deepen the knowledge of modern methods for solving combinatorial optimization problems.
- To deepen the knowledge of details of discrete models and other advanced applications of combinatorial optimization in financial optimization, resource optimization, and wider.

Transferable/Key Skills and other attributes:

- Direct applications in financial mathematics, economy, business, engineering, physics, and numerous other social and natural sciences.
- Competent mastering of the process of mathematical modelling and applications of the techniques of combinatorial optimization in problems from financial optimization, economics, and wider.
- Preparation of a detailed technical report or focused report paper describing a mathematical model of a specific mathematical problem the students encounter while investigation their possible future careers.

### **Metode poučevanja in učenja:**

- Pri predavanjih študentje spoznajo snov predmeta. Z uporabo obrnjenega (flipped) poučevanja na predavanjih aktivno spoznavajo povezavo med snovjo in njihovimi projekti.
- V okviru seminarских vaj študentje razumevanje snovi utrjujejo na večjem

### **Learning and teaching methods:**

- At the lectures, the students are familiarized with the course content. Applying flipped learning approach, they discuss their coursework projects in relation to the material of the course.
- At the tutorials, the student deepen their understanding of the material by working

projektu, povezanem z njihovo bodočo kariero. Razporejeni so v večje skupine, ki po metodah problemskega učenja obravnavajo izbrani problem od zbiranja podatkov, preko razvoja modela, izbora in prilagajanja ustreznih tehnoloških rešitev do razmisleka o implementaciji rešitve. Koncept poučevanja je podrobneje predstavljen kot ciljni aplikativni predmet.

on an extensive problem related to their future career. They are organized in larger groups who research the chosen problem guided by methodologies of problem-based learning. Within solving the problem, they experience all the stages from requirements and data gathering, model development, selecting and adapting technological solutions to discussing various aspects of implementation of the results.

**Načini ocenjevanja:**

**Assessment:**

Način (pisni izpit, ustno izpraševanje, naloge, projekt):	Delež (v %) / Weight (in %)	Type (examination, oral, coursework, project):
Seminarska naloga	80%,	Coursework report
Ustni izpit	20%	Oral exam
Vsaka izmed naštetih obveznosti mora biti opravljena s pozitivno oceno.		Each of the mentioned commitments must be assessed with a passing grade.
Pozitivna ocena pri seminarski nalogi je pogoj za pristop k izpitu.		Passing grade of the seminar exercise is required for taking the exam.

**Reference nosilca / Lecturer's references:**

1. BOKAL, Drago, CHIMANI, Markus, NOVER, Alexander, SCHIERBAUM, Jöran, STOLZMANN, Tobias, WAGNER, Mirko H., WIEDERA, Tilo. Properties of large 2-crossing-critical graphs. *Journal of graph algorithms and applications*. 2022, vol. 26, no. 1, str. 111-147. ISSN 1526-1719. DOI: [10.7155/jgaa.00585](https://doi.org/10.7155/jgaa.00585). [COBISS.SI-ID [144719363](https://www.cobiss.si/record/144719363)]
2. BOKAL, Drago, JEREBIC, Janja. Guarding a subgraph as a tool in pursuit-evasion games. *Discussiones mathematicae. Graph theory*. 2022, vol. 42, no. 1, str. 123-138. ISSN 1234-3099. [https://www.dmgt.uz.zgora.pl/publish/view\\_vol\\_article.php?E8F29296C3FB4B933A2E](https://www.dmgt.uz.zgora.pl/publish/view_vol_article.php?E8F29296C3FB4B933A2E). [COBISS.SI-ID [8147219](https://www.cobiss.si/record/8147219)]
3. SMOLE, Andreja, JAGRIČ, Timotej, BOKAL, Drago. Principal/Two-Agent model with internal signal. *Central European journal of operations research*. sep. 2021, vol. 29, no. 3, str. 791-808, ilustr. ISSN 1435-246X. <https://link.springer.com/content/pdf/10.1007/s10100-020-00719-0.pdf>, DOI: [10.1007/s10100-020-00719-0](https://doi.org/10.1007/s10100-020-00719-0). [COBISS.SI-ID [75068163](https://www.cobiss.si/record/75068163)]
4. VEGI KALAMAR, Alen, ŽERAK, Tadej, BOKAL, Drago. Counting Hamiltonian cycles in 2-tiled graphs. *Mathematics*. 2021, vol. 9, iss. 6, str. 1-27. ISSN 2227-7390. DOI: [10.3390/math9060693](https://doi.org/10.3390/math9060693). [COBISS.SI-ID [61574403](https://www.cobiss.si/record/61574403)]